QBronze Summary

n-qubit Quantum State $|\psi\rangle = \sum_{i=0}^{2^n-1} \alpha_i |i\rangle$, $\sum_{i=0}^{2^n-1} \alpha_i^2 = 1$, $\alpha_i \in \mathbb{R}$

Unitary Evolution $U|\psi\rangle = |\varphi\rangle = \sum_{i=0}^{2^n-1} \beta_i |i\rangle$, $\sum_{i=0}^{2^n-1} \beta_i^2 = 1$, $\beta_i \in \mathbb{R}$ $U^T U = \mathbb{1}$

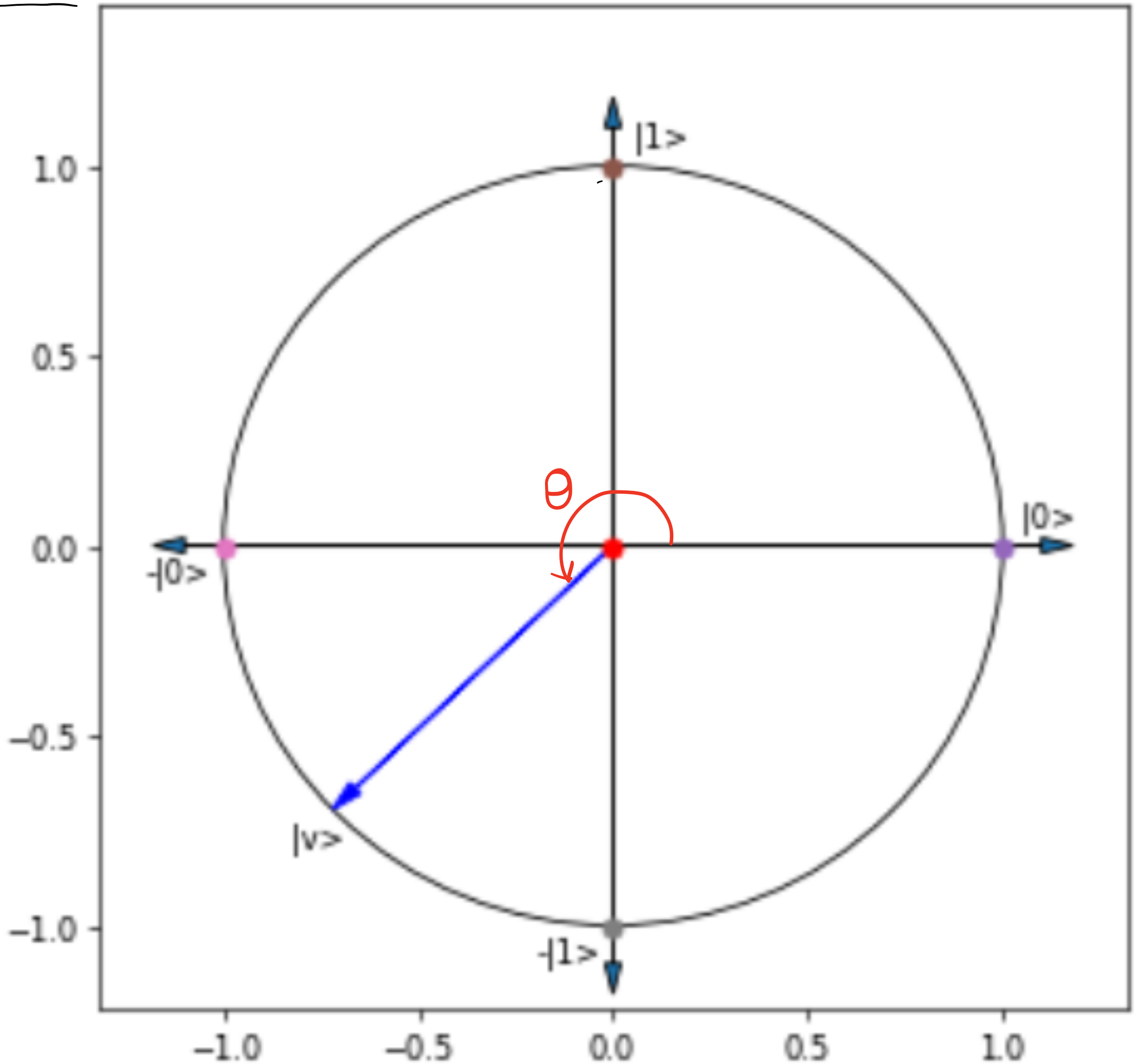
Measurement Probability to observe particular outcome i on measuring $|\psi\rangle$ is given by α_i^2

Qubit on a Unit Circle

$|\psi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$X|0\rangle = |1\rangle$

$X|1\rangle = |0\rangle$

$X|\psi\rangle = \cos\theta\, X|0\rangle + \sin\theta\, X|1\rangle$

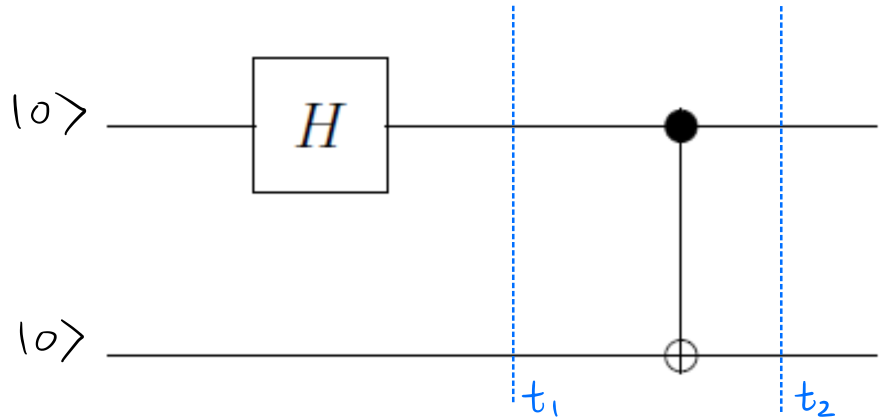
$= \cos\theta|1\rangle + \sin\theta|0\rangle$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$Z|0\rangle = |0\rangle$

$Z|1\rangle = -|1\rangle$

Preparing a Bell State



$$H|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) = |+\rangle$$

$$\begin{aligned} CNOT\,|00\rangle &= |00\rangle \\ |01\rangle &= |01\rangle \\ |10\rangle &= |11\rangle \\ |11\rangle &= |10\rangle \end{aligned}$$

$$|00\rangle \xrightarrow{H \otimes \mathbb{1}} |+\rangle|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)|0\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |10\rangle\right)$$

$$\xrightarrow{CNOT} \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) = |\varphi^+\rangle$$ a Bell state

<u>Entangled!</u>

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$

$\{|\varphi^+\rangle, |\varphi^-\rangle, |\psi^+\rangle, |\psi^-\rangle\}$

Entanglement vs Perfect Correlation

$$\hat{V} = \frac{1}{2}\begin{array}{cl}\begin{pmatrix}1\\0\\0\\1\end{pmatrix} & \begin{array}{l}00\\01\\10\\11\end{array}\end{array}$$

Perfect Classical Correlation

$$|\psi\rangle = \frac{1}{\sqrt{2}}\begin{array}{cl}\begin{pmatrix}1\\0\\0\\1\end{pmatrix} & \begin{array}{l}00\\01\\10\\11\end{array}\end{array}$$

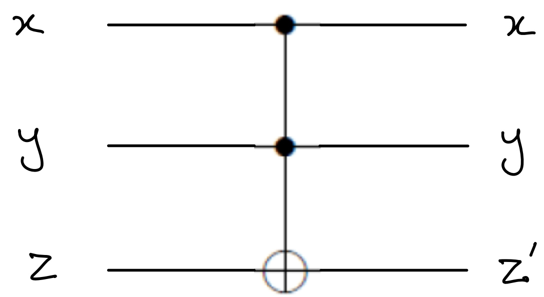
Maximally Entangled State

Toffoli Gate

$$\left.\begin{array}{l} |000\rangle \\ |001\rangle \\ |010\rangle \\ |011\rangle \\ |100\rangle \\ |101\rangle \end{array}\right\} \text{identity}$$

$$|110\rangle \longrightarrow |111\rangle$$

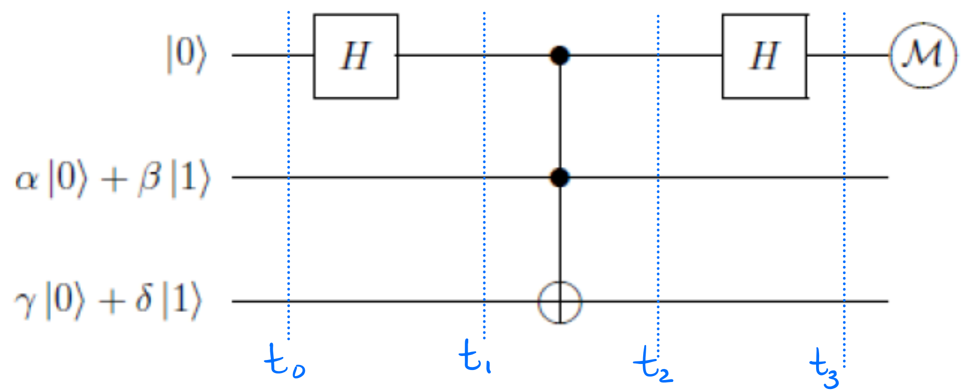
$$|111\rangle \longrightarrow |110\rangle$$



CCNOT

Toffoli

Circuit Evaluation



$$|0\rangle(\alpha|0\rangle+\beta|1\rangle)(\gamma|0\rangle+\delta|1\rangle)$$

$$\xrightarrow{H\otimes 1\otimes 1} \left(\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)\right)(\alpha|0\rangle+\beta|1\rangle)(\gamma|0\rangle+\delta|1\rangle)$$

$$= \frac{1}{\sqrt{2}}(\alpha|00\rangle+\beta|01\rangle+\alpha|10\rangle)(\gamma|0\rangle+\delta|1\rangle) + \frac{\beta}{\sqrt{2}}|11\rangle(\gamma|0\rangle+\delta|1\rangle)$$

$$\xrightarrow{CCNOT} \frac{1}{\sqrt{2}}(\alpha|00\rangle+\beta|01\rangle+\alpha|10\rangle)(\gamma|0\rangle+\delta|1\rangle) + \frac{\beta}{\sqrt{2}}|11\rangle(\gamma|1\rangle+\delta|0\rangle)$$

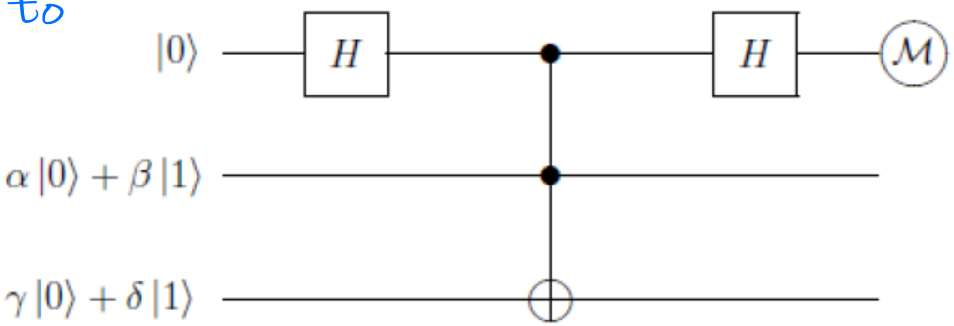
$$\xrightarrow{H\otimes 1\otimes 1} \frac{1}{2}(\alpha|00\rangle+\alpha|10\rangle+\beta|01\rangle+\beta|11\rangle+\alpha|00\rangle-\alpha|10\rangle)(\gamma|0\rangle+\delta|1\rangle) + \frac{1}{2}(\beta|01\rangle-\beta|11\rangle)(\gamma|1\rangle+\delta|0\rangle)$$

$$= \frac{1}{2}\Big[2\alpha\gamma|000\rangle+2\alpha\delta|001\rangle+\beta(\gamma+\delta)|010\rangle+\beta(\gamma+\delta)|011\rangle+\beta(\gamma-\delta)|110\rangle+\beta(\delta-\gamma)|111\rangle\Big]$$

Circuit Evaluation

What is probability for first qubit to be in state $|1\rangle$?

$$\frac{1}{2}\left(\beta^2(\gamma-\delta)^2\right)$$



Given that first qubit is measured in state $|1\rangle$, what is the probability distribution for second qubit?

with prob. 1, second qubit is in state 1.

Does there exist a choice for α, β, γ & δ for which first qubit is measured in state $|1\rangle$ with probability 1?

$$\alpha = 0,\ \beta = 1,\ \gamma = \frac{1}{\sqrt{2}},\ \delta = -\frac{1}{\sqrt{2}} \longrightarrow \frac{1}{2}\left(\beta^2(\gamma-\delta)^2\right) = 1$$

$$\frac{1}{2}\Big[2\alpha\gamma|000\rangle + 2\alpha\delta|001\rangle + \beta(\gamma+\delta)|010\rangle + \beta(\gamma+\delta)|011\rangle + \beta(\gamma-\delta)|110\rangle + \beta(\delta-\gamma)|111\rangle\Big]$$