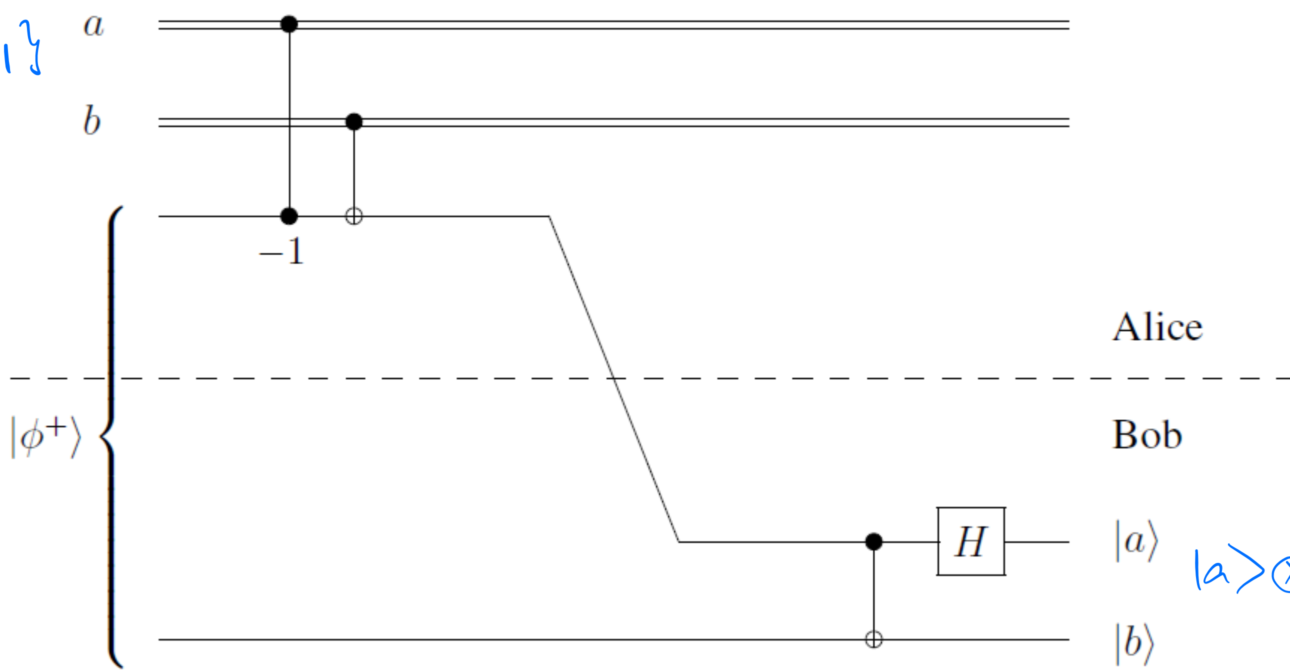
Superdense Coding

$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Holevo's Bound

$a, b \in \{0, 1\}$

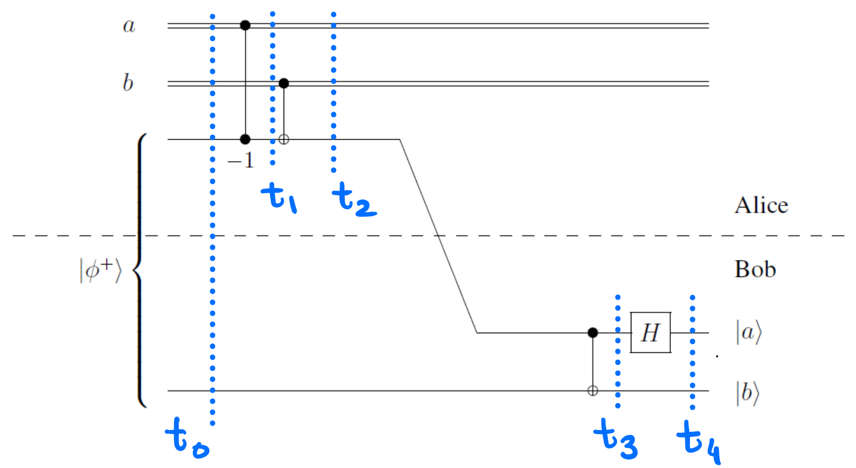
$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right) =$



$|a\rangle \otimes |b\rangle$

Superdense Coding

$$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

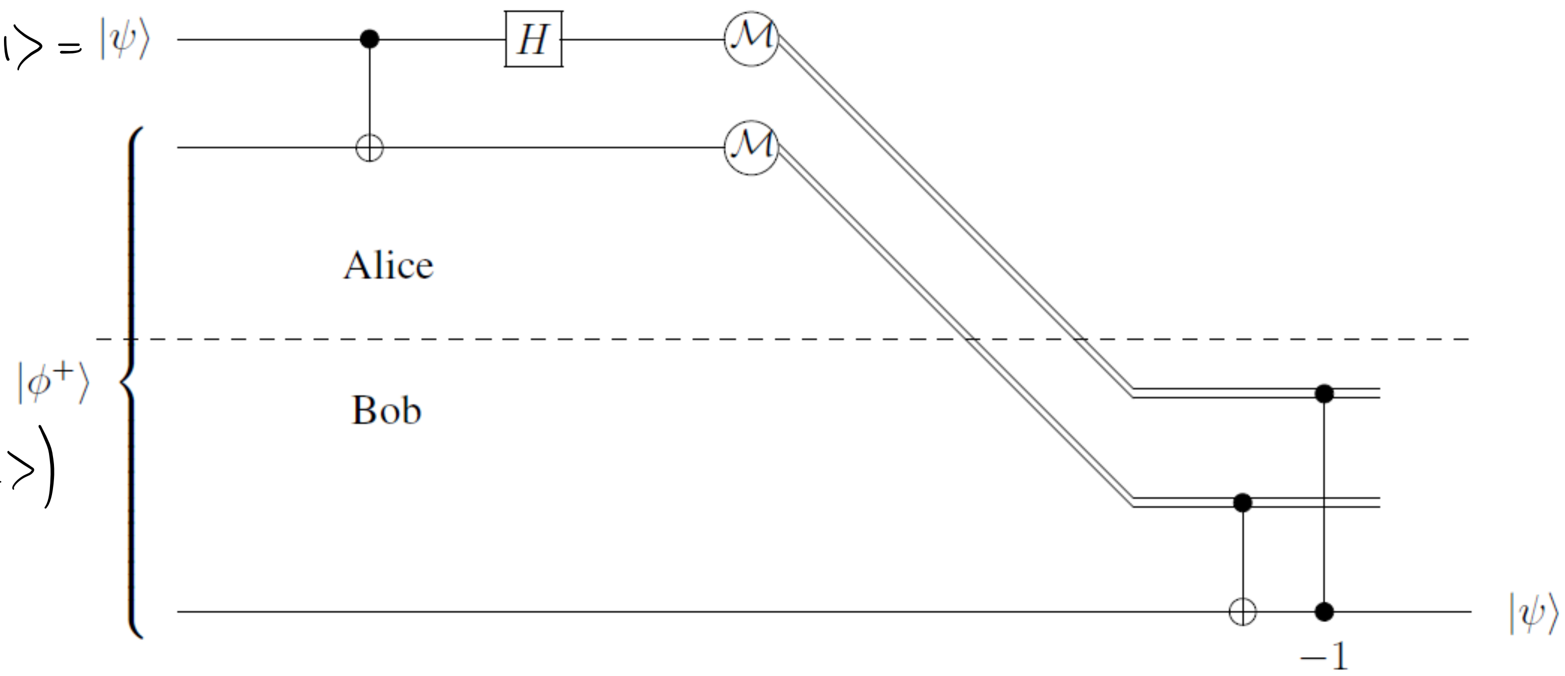


$$Z|0\rangle = |0\rangle$$
$$Z|1\rangle = -|1\rangle$$
$$H|0\rangle = |+\rangle$$
$$H|1\rangle = |-\rangle$$

ab	State at t_1	State at t_2	State at t_3	State at t_4
00	$\lvert\varphi^+\rangle$	$\lvert\varphi^+\rangle$	$\frac{1}{\sqrt{2}}\left(\lvert 00\rangle + \lvert 10\rangle\right) = \lvert +\rangle\lvert 0\rangle$	$\lvert 00\rangle$
01	$\lvert\varphi^+\rangle$	$\frac{1}{\sqrt{2}}\left(\lvert 10\rangle + \lvert 01\rangle\right)$	$\frac{1}{\sqrt{2}}\left(\lvert 11\rangle + \lvert 01\rangle\right) = \lvert +\rangle\lvert 1\rangle$	$\lvert 01\rangle$
10	$\frac{1}{\sqrt{2}}\left(\lvert 00\rangle - \lvert 11\rangle\right)$	$\frac{1}{\sqrt{2}}\left(\lvert 00\rangle - \lvert 11\rangle\right)$	$\frac{1}{\sqrt{2}}\left(\lvert 00\rangle - \lvert 10\rangle\right) = \lvert -\rangle\lvert 0\rangle$	$\lvert 10\rangle$
11	$\frac{1}{\sqrt{2}}\left(\lvert 00\rangle - \lvert 11\rangle\right)$	$\frac{1}{\sqrt{2}}\left(\lvert 10\rangle - \lvert 01\rangle\right)$	$\frac{1}{\sqrt{2}}\left(\lvert 1\rangle - \lvert 0\rangle\right)\lvert 1\rangle = -\lvert -\rangle\lvert 1\rangle$	$-\lvert 11\rangle$

Quantum Teleportation

$\alpha|0\rangle + \beta|1\rangle = |\psi\rangle$



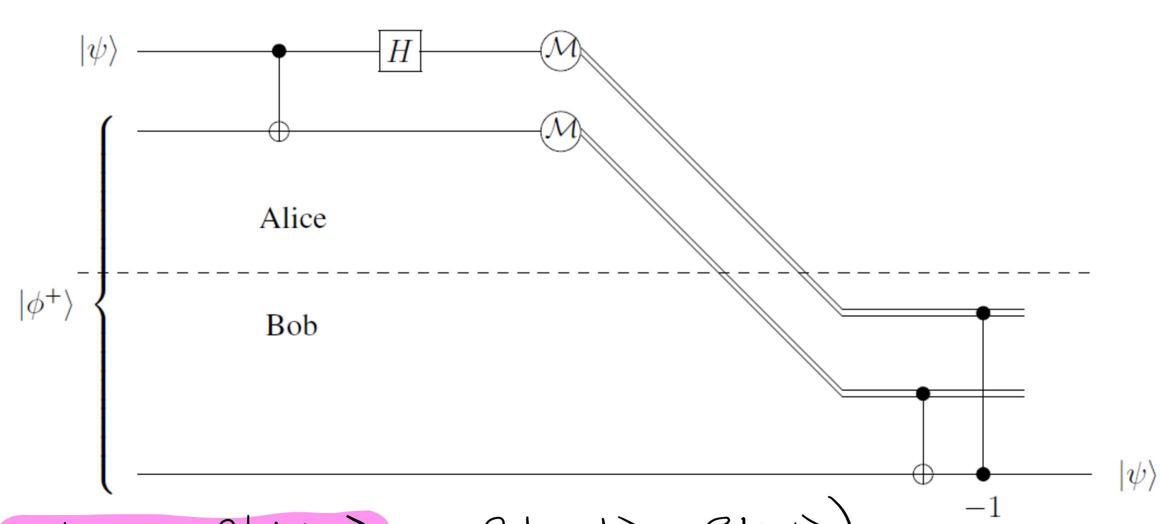
$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$

Quantum Teleportation

$$|\psi\rangle \otimes |\varphi^+\rangle = \left(\alpha|0\rangle + \beta|1\rangle\right)\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle\right)$$

$$\xrightarrow{CNOT\otimes\mathbb{1}} \frac{1}{\sqrt{2}}\left(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle\right)$$

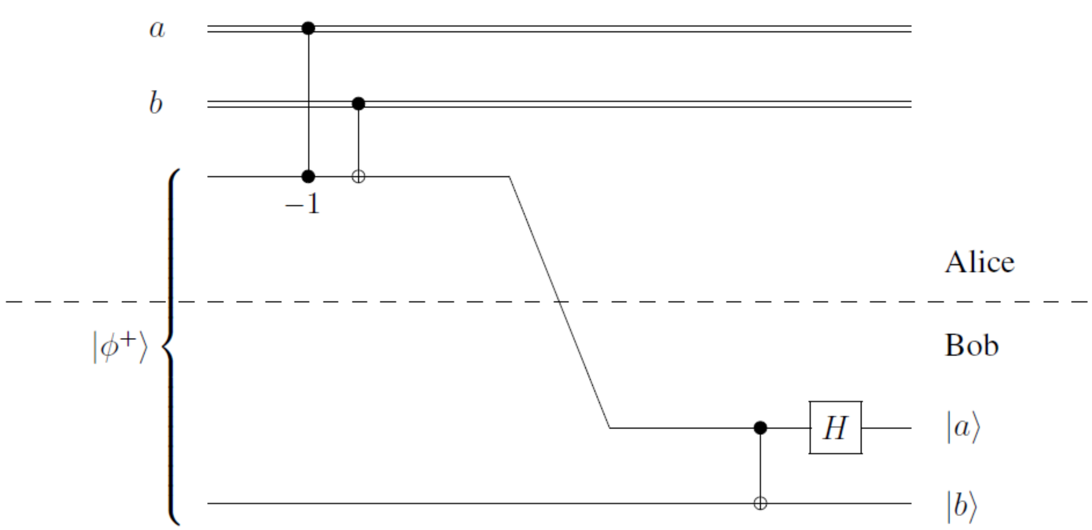


$$\xrightarrow{H\otimes\mathbb{1}\otimes\mathbb{1}}$$

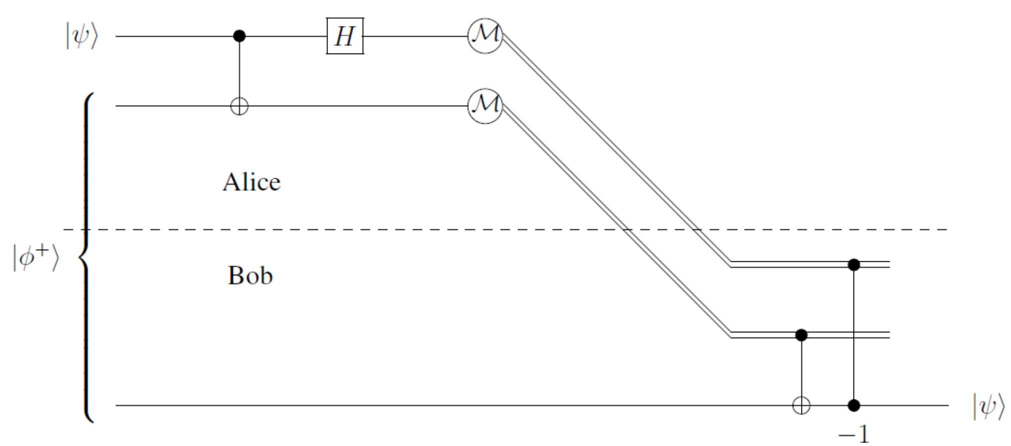
$$= \frac{1}{2}\left(\alpha|000\rangle + \alpha|100\rangle + \alpha|011\rangle + \alpha|111\rangle + \beta|010\rangle - \beta|110\rangle + \beta|001\rangle - \beta|101\rangle\right)$$

$$= \frac{1}{2}\left(|00\rangle\underbrace{\left(\alpha|0\rangle + \beta|1\rangle\right)}_{\substack{\mathbb{1} \\ |\psi\rangle}} + |01\rangle\underbrace{\left(\alpha|1\rangle + \beta|0\rangle\right)}_{\substack{X(\alpha|1\rangle + \beta|0\rangle) \\ |\psi\rangle = \alpha|0\rangle + \beta|1\rangle}} + |10\rangle\underbrace{\left(\alpha|0\rangle - \beta|1\rangle\right)}_{\substack{Z(\alpha|0\rangle - \beta|1\rangle) \\ = \alpha|0\rangle + \beta|1\rangle}} + |11\rangle\underbrace{\left(\alpha|1\rangle - \beta|0\rangle\right)}_{\substack{ZX(\alpha|1\rangle - \beta|0\rangle) \\ Z(\alpha|0\rangle - \beta|1\rangle) \\ \alpha|0\rangle + \beta|1\rangle}}\right)$$

Comparison



a
b
-1
Alice
Bob
$|\phi^+\rangle$
H
$|a\rangle$
$|b\rangle$



$|\psi\rangle$
H
$\mathcal{M}$
$\mathcal{M}$
Alice
Bob
$|\phi^+\rangle$
$|\psi\rangle$
-1