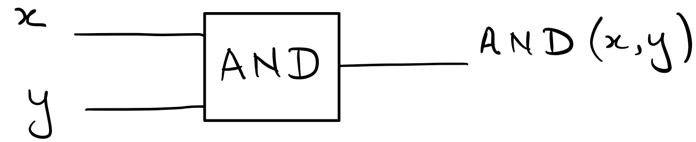
QWORLD

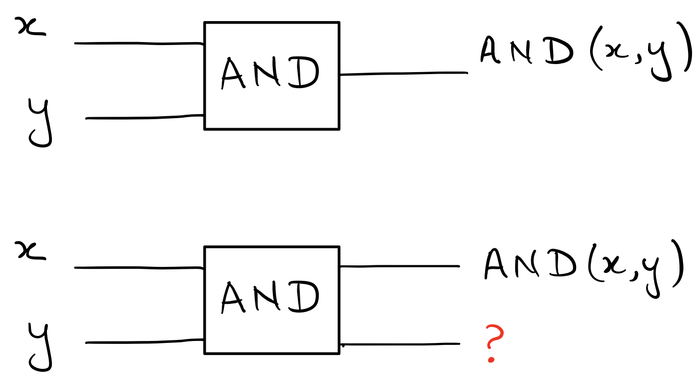
INTRODUCTION TO QUANTUM ALGORITHMS

Jibran Rashid

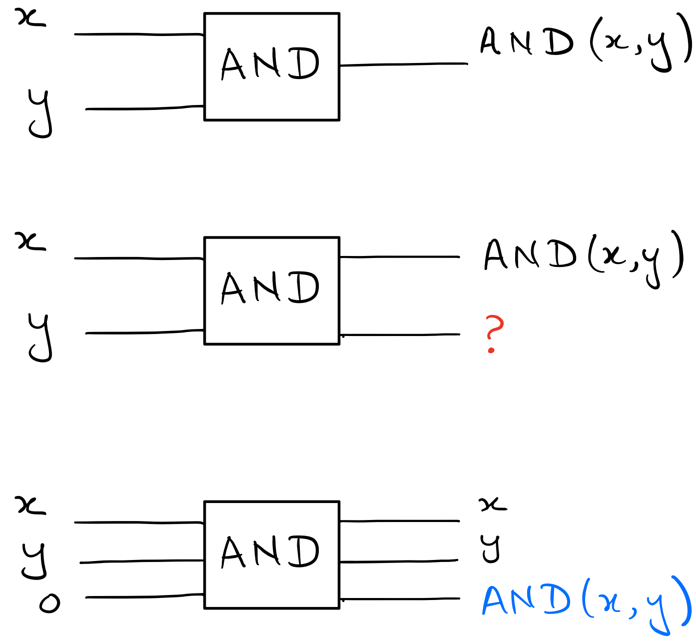
Reversible Transformations



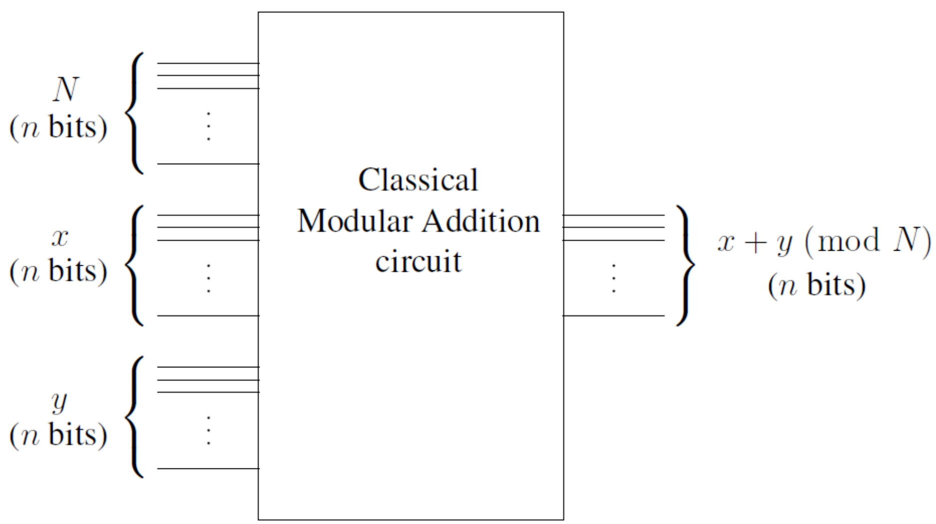
Reversible Transformations

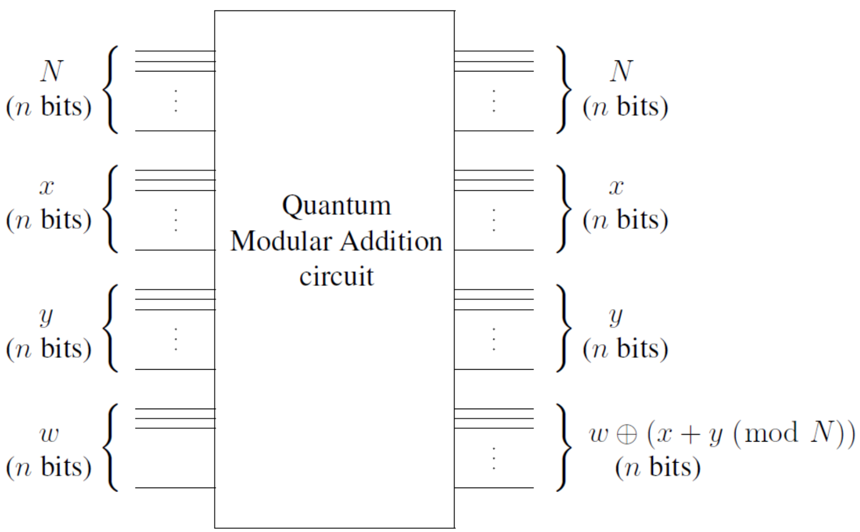


Reversible Transformations



Reversible Computation





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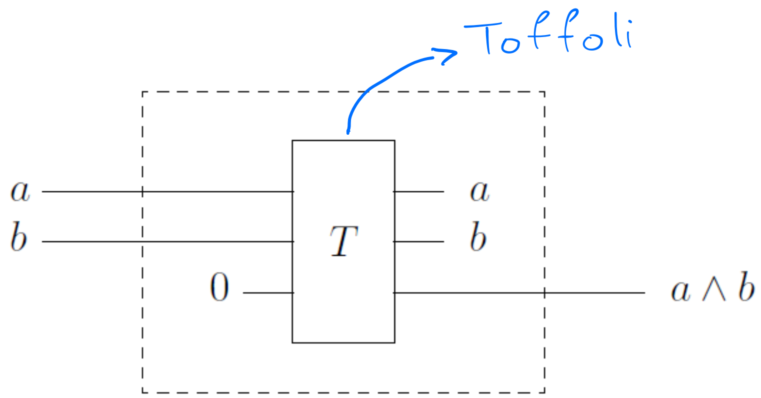

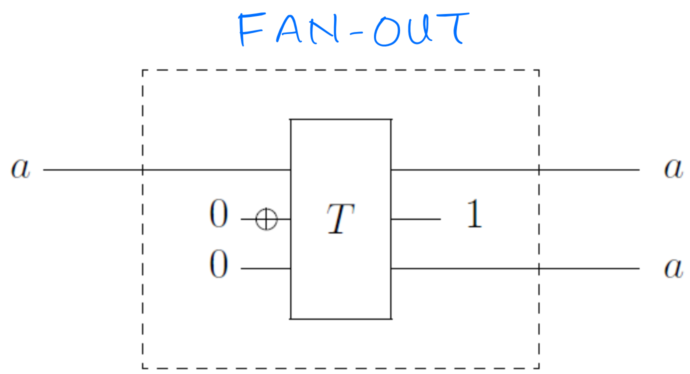

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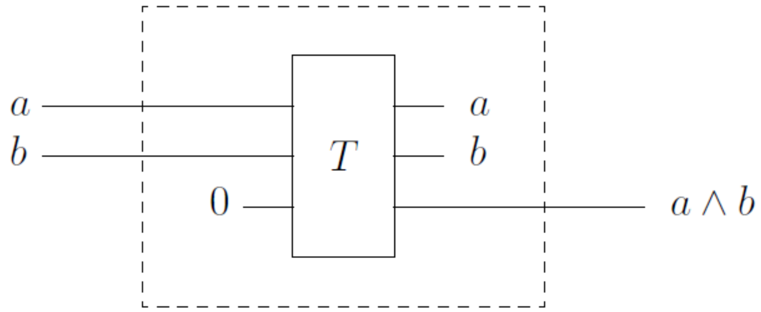
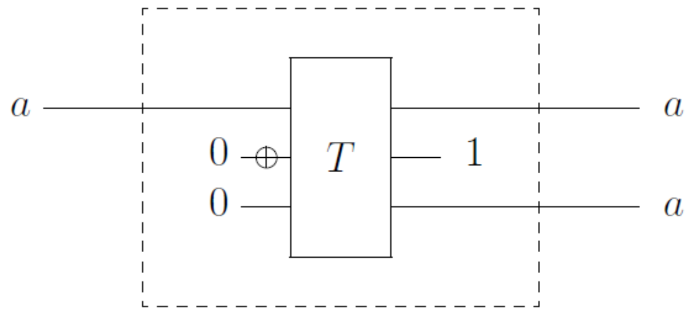
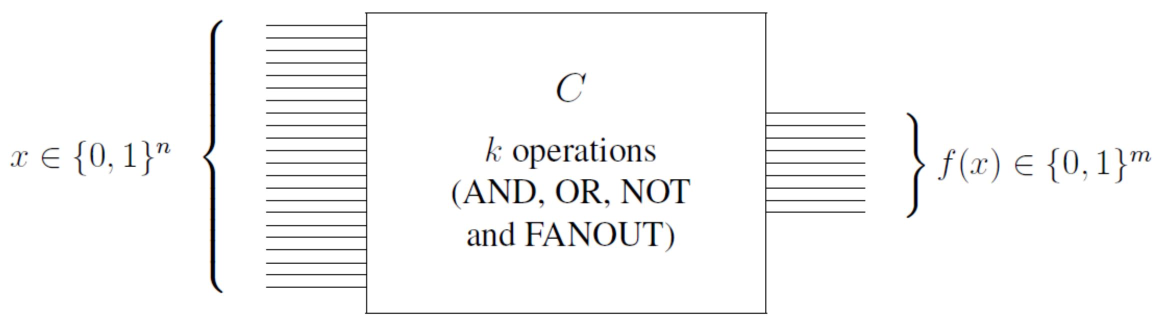
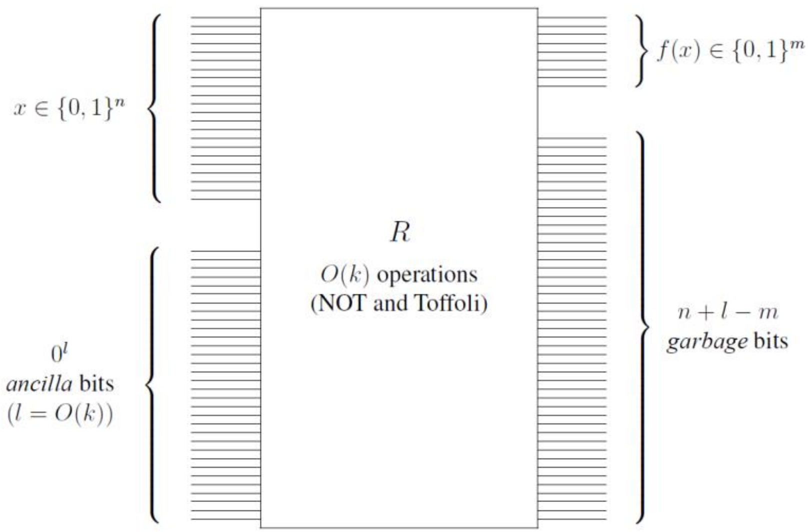
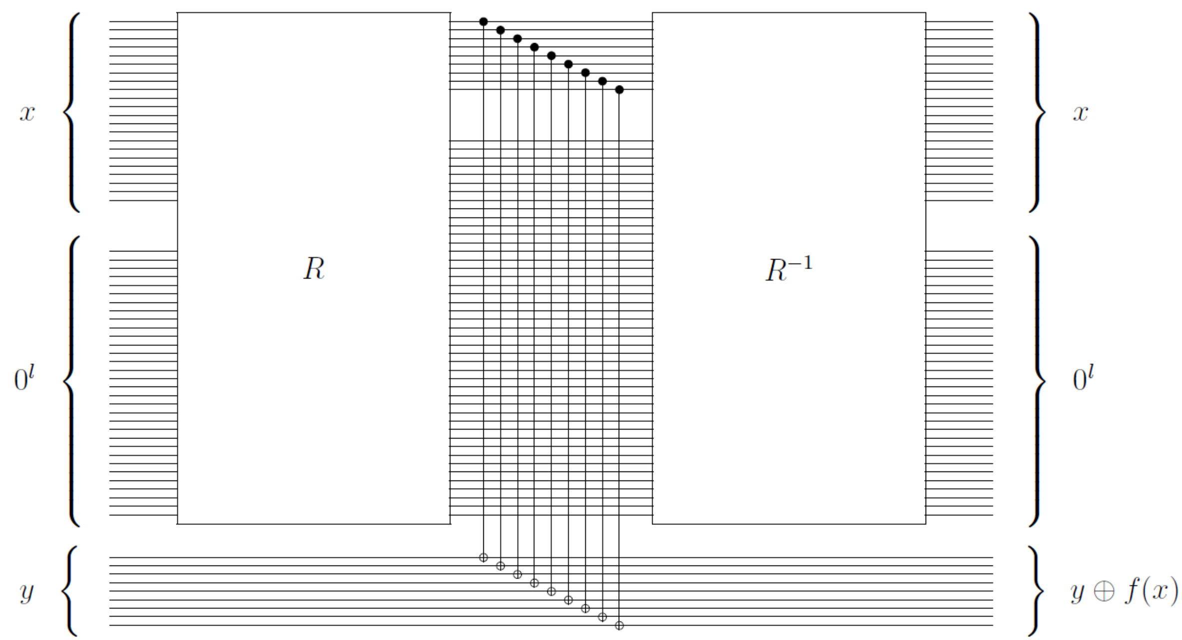




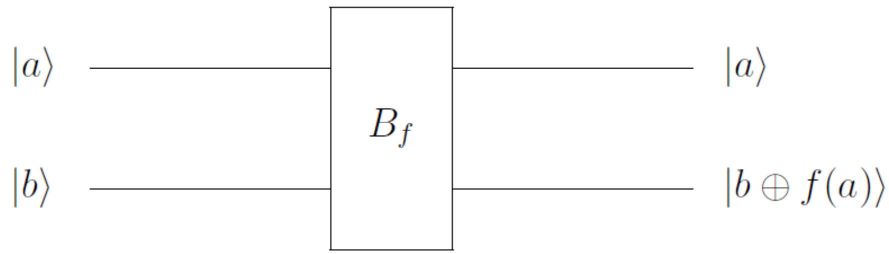


Reversible Computation





Classical Gates Via Unitaries



$a, b \in \{0,1\}$

$B_f : |a\rangle|b\rangle \longrightarrow |a\rangle|b \oplus f(a)\rangle$

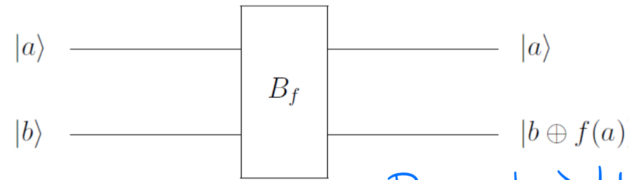
$f : \{0,1\}^n \longrightarrow \{0,1\}$

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

Phase Kickback

Recall

$|-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$



$B_f : |a\rangle|b\rangle \longrightarrow |a\rangle|b \oplus f(a)\rangle$

Consider $B_f|a\rangle|-\rangle$

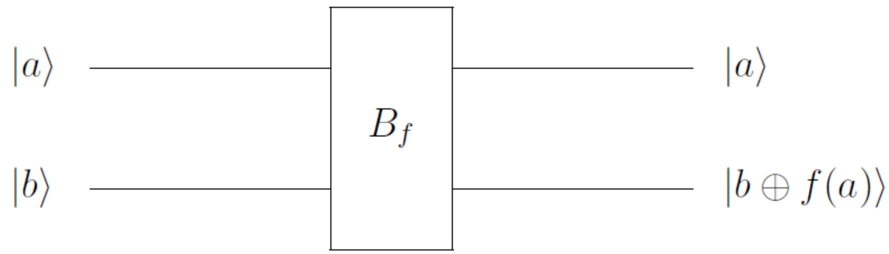
$$= B_f|a\rangle \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) = \frac{1}{\sqrt{2}}\left(B_f|a\rangle|0\rangle - B_f|a\rangle|1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(|a\rangle|0 \oplus f(a)\rangle - |a\rangle|1 \oplus f(a)\rangle\right)$$

$$f(a) = \left\{\begin{array}{ll} 0 \longrightarrow \frac{1}{\sqrt{2}}|a\rangle\left(|0\rangle - |1\rangle\right) \\ 1 \longrightarrow \frac{1}{\sqrt{2}}|a\rangle\left(|1\rangle - |0\rangle\right) \end{array}\right\} = (-1)^{f(a)}|a\rangle|-\rangle$$

$$B_f : |a\rangle|-\rangle \longrightarrow (-1)^{f(a)}|a\rangle|-\rangle$$

Classical Gates Via Unitaries



$$B_f : |a\rangle|-\rangle \longrightarrow (-1)^{f(a)}|a\rangle|-\rangle$$

QWORLD

QUANTUM QUERY ALGORITHMS

Jibran Rashid

Simple Search

f_{00}

input	output
00	1
01	0
10	0
11	0

f_{01}

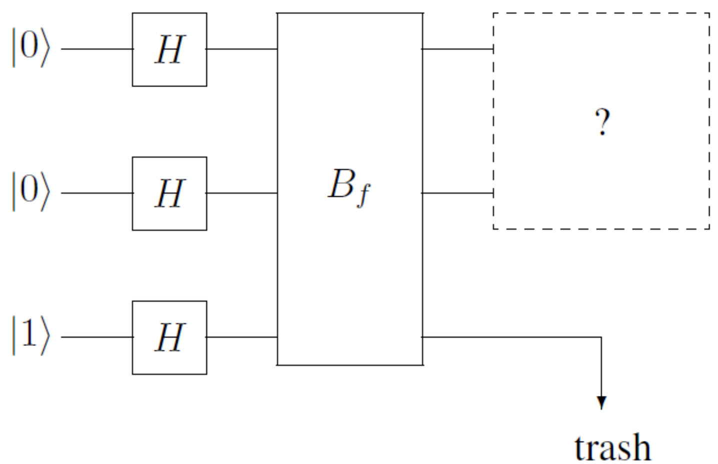
input	output
00	0
01	1
10	0
11	0

f_{10}

input	output
00	0
01	0
10	1
11	0

f_{11}

input	output
00	0
01	0
10	0
11	1



Hadamard on n Qubits

$$H|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right), \quad H|1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

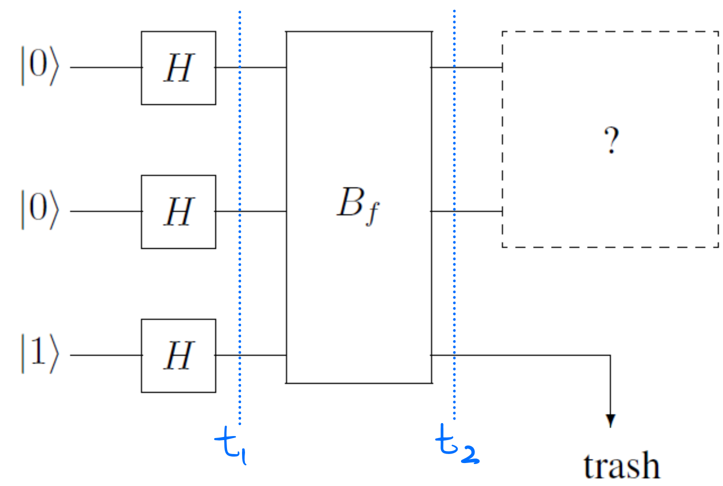
$$H \otimes H\,|00\rangle = H|0\rangle \otimes H|0\rangle = |+\rangle|+\rangle$$

$$= \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

Simple Search

$$|\psi_1\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

Determine $|\psi_2\rangle$ via phase kickback

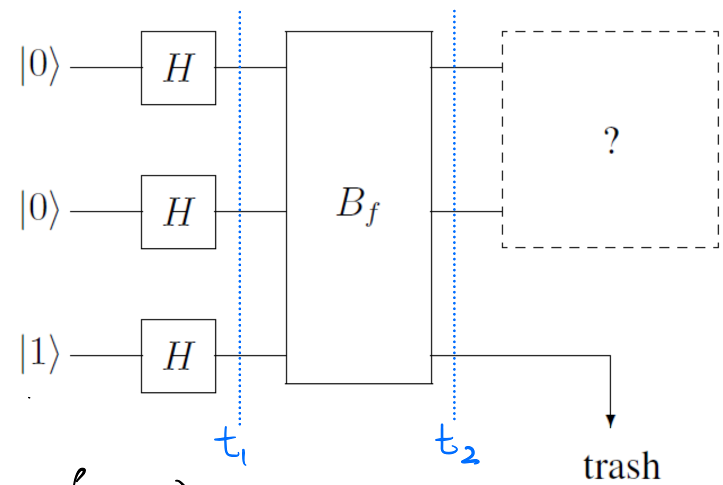


Simple Search

$$|\psi_1\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

Determine $|\psi_2\rangle$ via phase kickback

$$B_f|a\rangle|-\rangle \longrightarrow (-1)^{f(a)}|a\rangle|-\rangle$$



$$B_f|\psi_1\rangle = \frac{1}{2}\left((-1)^{f_{00}}|00\rangle + (-1)^{f_{01}}|01\rangle + (-1)^{f_{10}}|10\rangle + (-1)^{f_{11}}|11\rangle\right)|-\rangle$$

Simple Search

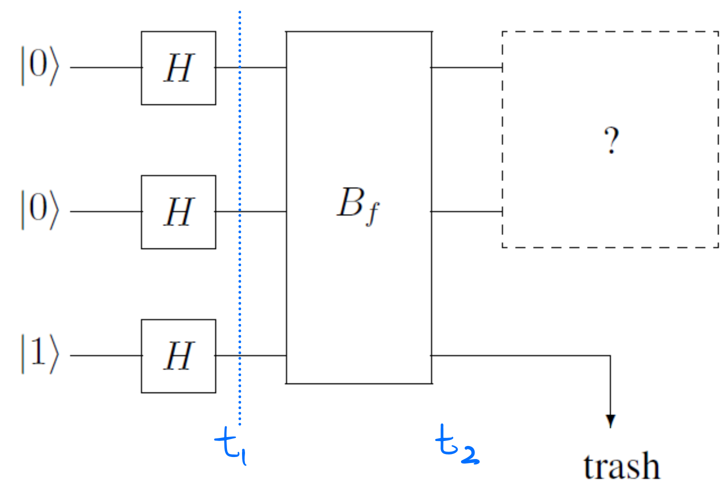
$$\frac{1}{2}\left((-1)^{f(00)}|00\rangle + (-1)^{f(01)}|01\rangle + (-1)^{f(10)}|10\rangle + (-1)^{f(11)}|11\rangle\right)$$

$$f = f_{00} \Rightarrow \frac{1}{2}\left(-|00\rangle + |01\rangle + |10\rangle + |11\rangle\right) \longrightarrow |00\rangle$$

$$f = f_{01} \Rightarrow \frac{1}{2}\left(+|00\rangle - |01\rangle + |10\rangle + |11\rangle\right) \longrightarrow |01\rangle$$

$$f = f_{10} \Rightarrow \frac{1}{2}\left(+|00\rangle + |01\rangle - |10\rangle + |11\rangle\right) \longrightarrow |10\rangle$$

$$f = f_{11} \Rightarrow \frac{1}{2}\left(+|00\rangle + |01\rangle + |10\rangle - |11\rangle\right) \longrightarrow |11\rangle$$



Hadamard on n Qubits

$$H|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right), \quad H|1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

$a \in \{0, 1\}$

$$H|a\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + (-1)^a \underline{|1\rangle}\right)$$

updated: Session video has a typo here!

$$H\left(H|a\rangle\right) = H\left(\frac{1}{\sqrt{2}}\left(|0\rangle + (-1)^a |1\rangle\right)\right)$$

$$H\left(\frac{1}{\sqrt{2}}\left(|0\rangle + (-1)^a |1\rangle\right)\right) = |a\rangle$$

Deutsch's Algorithm

$|\psi_0\rangle = |01\rangle$

$|\psi_1\rangle = H \otimes H \, |\psi_0\rangle = |+\rangle|-\rangle$

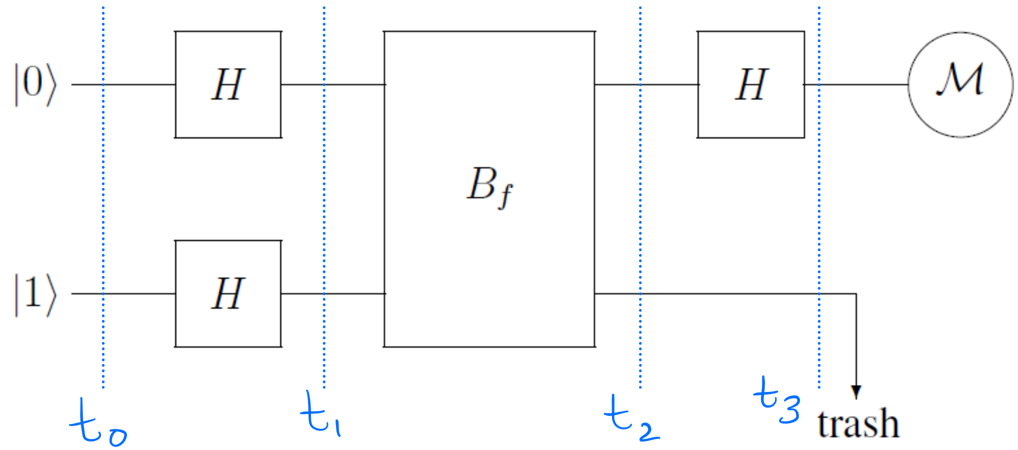
$B_f : |a\rangle|-\rangle \longrightarrow (-1)^{f(a)}|a\rangle|-\rangle$

$|\psi_2\rangle = B_f \, |\psi_1\rangle$

$= \frac{1}{\sqrt{2}}\left(B_f|0\rangle|-\rangle + B_f|1\rangle|-\rangle\right)$

$= \frac{1}{\sqrt{2}}\left((-1)^{f(0)}|0\rangle|-\rangle + (-1)^{f(1)}|1\rangle|-\rangle\right)$

$= \frac{(-1)^{f(0)}}{\sqrt{2}}\left(|0\rangle + (-1)^{f(1)\oplus f(0)}|1\rangle\right)|-\rangle$



constant

		f_0	f_1	f_2	f_3
Input	0	0	0	1	1
	1	0	1	0	1

Want to know

Is f constant: f_0, f_3

OR

balanced: f_1, f_2

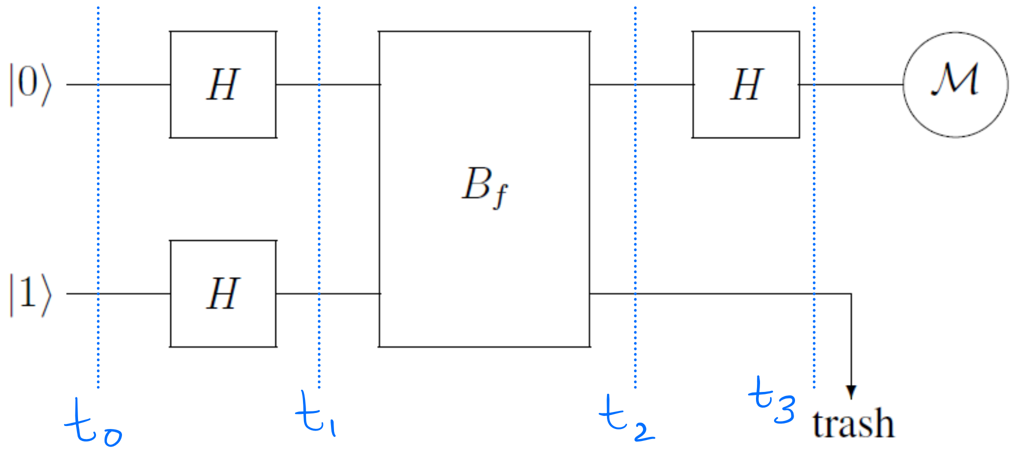
Classically: 2 Queries

Deutsch's Algorithm

$$H\left(\frac{1}{\sqrt{2}}\left(|0\rangle + (-1)^{a}|1\rangle\right)\right) = |a\rangle$$

$$\frac{(-1)^{f(0)}}{\sqrt{2}}\left(|0\rangle + (-1)^{\overbrace{f(1)\oplus f(0)}^{a}}|1\rangle\right)|-\rangle$$

$$\frac{(-1)^{f(0)}}{\sqrt{2}}\left(|0\rangle + (-1)^{a}|1\rangle\right)|-\rangle$$



$$|\psi_3\rangle = H\otimes 1|\psi_2\rangle = |a\rangle|-\rangle = |f(0)\oplus f(1)\rangle|-\rangle$$

Input		f_0	f_1	f_2	f_3
	0	0	0	1	1
	1	0	1	0	1

Constant $\longrightarrow$ $f(0)\oplus f(1) = 0$

Balanced $\longrightarrow$ $f(0)\oplus f(1) = 1$

$f(0)$	$f(1)$	$f(0)\oplus f(1)$
0	0	0
0	1	1
1	0	1
1	1	0